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**Efektywność i bezpieczeństwo gospodarowania**

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# SOME METHODS OF EVALUATION TRANSACTION COSTS

**Abstract**: In this article transaction costs are considered. Some investment company invest money in different projects with different interest rates and number of year. But real interest rate is another, because company hired consulting firm and paid a commission. The purpose of this work is evaluation transaction costs in investment sphere.

**Keywords:** transaction costs, interest rates, trading losses, real interest rate

Nowadays, transaction costs become very popular. In the most general form, transaction costs are resources, which lost to the committed transactions. Production costs, according to the new institutional economics, consist of two parts – the transformation costs that are connected with the change or reproduction of the physical characteristics of goods, and transaction costs, reflecting the change, or legal play, and generally in more – institutional characteristics. When present the economy as a life-support system, then the transaction costs can be considered as an expense of the economic system.

For explanation of the phenomenon of the most significant transaction costs there are two ways: the discrepancy between the economic interests of agents that interact with each other and the phenomenon of uncertainty. Uncertainty is determined not only by distortion of information, which have the individuals, but also limit the possibility of mastering agents that owns it.

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Given that, there are two aspects in explaining of the transaction costs, they can be defined as the costs of coordinating the activities of economic agents and distribution of relief from the conflict between them. Since coordination – a key component of any organization, excluding transaction costs economic analysis would be little productive.

If transaction costs are zero, the resources are divided and used there, where they are most valuable (if not to take into account the effect of income), regardless of the initial distribution of property rights between economic agents.

Thus, in recent decades, transaction costs have become very important in the investment process. Firstly – it is indirect costs that are not taxed, but they are present at almost all investment processes, so their total amount is quite large, and it is not involved in the direct investment, but they have some affects in the investment climate. Known Coase theorem states, that investment is most effective if transaction costs are equal zero. Thus, their evaluation is important and actual.

Let the investment company invests some money in $k$-investment projects. In the first project company invest $X_1$ with interest rate $r_1$ for $m_1$ years; in the second – $X_2$ with interest rate $r_2$ for $m_2$ years; in the $k$ project invest $X_k$ with interest rate $r_k$ for $m_k$ years.

Company wants to be sure that the money will be returned. So for each project company hires a consulting firm, which check the solvency of selected projects.

After paying commissions company real will invest $(X_1 - x_1)$ in the first project, $(X_k - x_k)$ – in $k$ project respectively. If the investment company is not confident in the reliability testing, they hired other consulting firms and they also paid a commission.

Thus, the transaction costs for the first project $x_{11}, x_{12}, \dots, x_{1m}$ depending on the number of consulting firm. Therefore, the total transaction costs for the first project equal $x_1 = x_{11} + x_{12} + \dots + x_{1m}$, for $k$ project $x_k = x_{k1} + x_{k2} + \dots + x_{km}$. The number of consulting firm can be different for every investment company. Without transaction costs future earnings for the first project is $X_1(1 + r_1)^{m_1}$, for the $k$ project –$X_k(1 + r_k)^{m_k}$. Accordingly, given the commission, the amount is reduced. For the first project $(X_1 - x_1)(1 + \Delta_1)^{m_1}$, where $\Delta$ – new interest rate, for $k$ project- $(X_k - x_k)(1 + \Delta_k)^{m_k}$. In this case, $\Delta$ is a real interest rate.

Now, we write the equation of equivalence for each project:

$$(X_1 - x_1)(1 + \Delta_1)^{m_1} = X_1(1 + r_1)^{m_1}$$

$$(X_2 - x_2)(1 + \Delta_2)^{m_2} = X_2(1 + r_2)^{m_2}$$

And for the $k$ project

$$(X_k - x_k)(1 + \Delta_k)^{m_k} = X_k(1 + r_k)^{m_k}.$$

Then we can find real interest rate:

$$\frac{(1+\Delta_1)^{m_1}}{(1+r_1)^{m_1}} = \frac{X_1}{(X_1 - x_1)}$$

$$\frac{1+\Delta_1}{1+r_1} = \left(\frac{X_1}{X_1 - x_1}\right)^{\frac{1}{m_1}}$$

$$\Delta_1 = \left(\frac{X_1}{X_1 - x_1}\right)^{\frac{1}{m_1}} (1 + r_1) - 1$$

Resiprocally, for the second project

$$\Delta_2 = \left(\frac{X_2}{X_2 - x_2}\right)^{\frac{1}{m_2}} (1 + r_2) - 1$$

And for $k$ project

$$\Delta_k = \left(\frac{X_k}{X_k - x_k}\right)^{\frac{1}{m_k}} (1 + r_k) - 1.$$

Let $\Delta = \min\{\Delta_1, \Delta_2, \dots, \Delta_k\}$. Then the future earning is equal

$$Y = \sum_{i=1}^{k} (X_i - x_i)(1 + \Delta)^{m_i}.$$

So, we find minimal interest rate, but we can have some projects with this interest rate. Suppose, we have *l*, *l*<*k*, projects with this property.Thus, we need to find the investment project, in which costs are invested no more than $m_0$ years. Future earnings for the *l* project is equal

$$Y_l = (X_l - x_l)(1 + \Delta)^{m_l}.$$

So, we can find $m_l$.

$$\frac{Y_l}{X_l - x_l} = (1 + \Delta)^{m_l}$$

$$\ln \frac{Y_l}{X_l - x_l} = m_l \ln(1 + \Delta_l)$$

$$m_l = \frac{\ln \frac{Y_l}{X_l - x_l}}{\ln(1 + \Delta)}$$

$$m_l = \log_{(1+\Delta)} \frac{Y_l}{X_l - x_l}.$$

In this case $m_0 = \max\left\{\log_{(1+\Delta)} \frac{Y_1}{X_1 - x_1}; \log_{(1+\Delta)} \frac{Y_2}{X_2 - x_2}; \dots; \log_{(1+\Delta)} \frac{Y_l}{X_l - x_l}\right\}$.

If the investment company investing money in projects no more than $m_0$ years, then the future earning for each project can be calculated as follows:

$$Y_1 = (X_1 - x_1)(1 + \Delta)^{m_0}$$

$$Y_2 = (X_2 - x_2)(1 + \Delta)^{m_0}$$

And for $k$ project

$$Y_l = (X_l - x_l)(1 + \Delta)^{m_0}$$

So, the total earning of investment company is

$$Y = (X_1 - x_1)(1 + \Delta)^{m_0} + \dots + (X_l - x_l)(1 + \Delta)^{m_0} = (1 + \Delta)^{m_0} \sum_{i=1}^{l} (X_i - x_i).$$

After this we can find investment no more than $m_0$years with minimal variance:

$$EY = E(Y_1 + Y_2 + \dots + Y_k) = EY_1 + EY_2 + \dots + EY_k$$

$$\frac{EY_1 + EY_2 + \dots + EY_k}{k} \xrightarrow{\text{звч}} \frac{Y_1 + Y_2 + \dots + Y_k}{k}$$

$$EY = (X_1 - x_1)(1 + \Delta_1)^{m_0} + (X_2 - x_2)(1 + \Delta_2)^{m_0} + \dots + (X_1 - x_1)(1 + \Delta_1)^{m_0}$$

$$= \sum_{i=1}^{\kappa} (X_i - x_i)(1 + \Delta_i)^{m_0}$$

$$EY^2=\frac{1}{k}\sum_{i=1}^{K}(X_i - x_i)^2(1+\Delta_i)^{2m_0}$$

$$DY=EY^2 - (EY)^2$$

$$DY = \frac{1}{k}\sum_{i=1}^{K}(X_i - x_i)^2(1+\Delta_i)^{2m_0} - (\sum_{i=1}^{K}(X_i - x_i)(1+\Delta_i)^{m_0})^2$$

$$min\, DY=\frac{1}{k}\min_{m_0}\sum_{i=1}^{K}(X_i - x_i)^2(1+\Delta_i)^{2m_0} - (\sum_{i=1}^{K}(X_i - x_i)(1+\Delta_i)^{m_0})^2$$

Consider an example.

Let investment company invest money in 5 investment projects. In the first 150 with interest rate 7% for 8 years, in the second 200 with 9% for 7 years, in the third 180 with 11% for 5 years, in the fourth 240 with 10% for 8 years and in the fifth 130 with 8% for 10 years. For this projects transaction costs equal 30, 45, 25, 35 and 18 accordingly.

So, we need to choose project with minimal interest rate. For this we need to calculate real interest rate. We used next equation:

$$\Delta_k = \left(\frac{X_k}{X_k - x_k}\right)^{\frac{1}{m_k}}(1+r_k)-1$$

From this, $\Delta_1$=10%, $\Delta_2$=13%, $\Delta_3$=14%, $\Delta_4$=12%, $\Delta_5$=10%.

So, minimal real interest rates are observed in the first and fifth projects.

In this situation we need to choose project, in which company investsmoney for more years. It is project number five.

Lets consider another situation with *n* investment ptoject.

$X_1$ – investment in the first project for$k_1$ – years with interest rate $\rho_1$,$X_n$ – investment in the *n* project for $k_n$ – years with interest rate $\rho_n$.

We write these in the following matrix

$$\begin{pmatrix} X_1 & X_2 & \dots & X_n \\ k_1 & k_2 & \dots & k_n \\ \rho_1 & \rho_2 & \dots & \rho_n \end{pmatrix}$$

In order to ensure the solvency of the customers investment companies sent them investment experts, and their services are charged on the investment project, and impact on the interest rates. Consider, that the average cost for experts in these projects, are $x_1$, $x_2$,…, $x_n$. If this expert group is not satisfied with their conclusions of investors then investors sent another group of experts and so on. Suppose that the probability of confidence in the results of expert opinions on the projects is equal $p_1$,…, $p_n$ respectively. And the maximum number of expert groups in this case is equal $r_1, r_2$, ….

Suppose that a random variable $x_1$ has a binomial distribution with parameters $p_1$ and $r_1$. So, $x_1 \sim$ Bi($p_1, r_1$), respectively $x_n \sim$ Bi($p_n, r_n$).

Thus, the expectation E$x_i$= $p_i r_i$ and the variance D$x_i$= $r_i p_i(1 - p_i)$.

Now, we can find project with minimal interest rate. Firstly, we write matrix with information about investment project without transaction costs.

$$\begin{pmatrix} X_1 - x_1 & X_2 - x_2 & \dots & X_n - x_n \\ k_1 & k_2 & \dots & k_n \\ \rho_1^* & \rho_2^* & \dots & \rho_n^* \end{pmatrix}.$$

The future earnings is equal

$$Y_n = (X_n - x_n)(1 + \rho_n^*)^{k_n}$$

Find the interest rate:

$$\rho_n^* = \left(\frac{Y_n}{X_n - x_n}\right)^{\frac{1}{k_n}} - 1$$

So, the minimal interest rate is

$$\mathrm{P}^* = \min\left\{\left(\frac{Y_n}{X_1 - x_1}\right)^{\frac{1}{k_1}} - 1; \dots ; \left(\frac{Y_n}{X_n - x_n}\right)^{\frac{1}{k_n}} - 1\right\}.$$

Thus, $\mathrm{Y} = \sum_{\mathrm{i}=1}^{k} (X_{\mathrm{i}} - x_{\mathrm{i}})(1 + \rho *)^{m_{\mathrm{i}}}$.

Consider another distribution. Suppose, $x_n \sim \Gamma(p_n, r_n)$. Then expectation $\mathrm{E}x_n = \frac{p_n}{r_n}$ and $\mathrm{D}x_n = \frac{p_n}{r_n{}^2}$. Average costs for experts $\frac{p_n}{r_n} x_n$ and standard deviation is equal $\frac{\sqrt{p_n}}{r_n} x_n$.

Now, let $x_n \sim \mathrm{R}(a_n, b_n)$. In this case $\mathrm{E}x_n = \frac{a_n + b_n}{2}$, $\mathrm{D}x_n = \frac{(b_n - a_n)^2}{12}$. Average costs for experts $\frac{a_n + b_n}{2} x_n$ and standard deviation $\frac{b_n - a_n}{2\sqrt{3}} x_n$.

So, the evaluation of transaction costs is possible in different ways. This is very important and actual problem in our economics.

## Bibliography


Єлейко Я., Куликівець Н., Зброїнська Б., *Моделювання трансакційних витрат і умовах невизначеності*, Вісник ЛНУ Сер. прикл. мат. та інформ.. 2012, вип. 18. p. 222-228.


## Abstrakt

### Wybrane metody oceny kosztów transakcyjnych


Celem artykułu jest wycena kosztów transakcyjnych w sferze inwestycji finansowych. Dokonano jej za pomocą modelu matematycznego. Fundusze inwestycyjne lokują środki w rozmaite projekty różniące się stopą procentową i okresem inwestycji. Ocena opłacalności inwestycji powinna uwzględniać nominalną, a nie realną stopę procentową. Realna stopa procentowa jest niższa od nominalnej. Pomniejszają ją koszty prowizji wypłacanych firmom consultingowym zatrudnianym przez fundusze.



**Słowa kluczowe:** koszty transakcyjne, stopy procentowe, transakcyjne straty, realna stopa procentowa