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Od czasów Adamieckiego do współczesności

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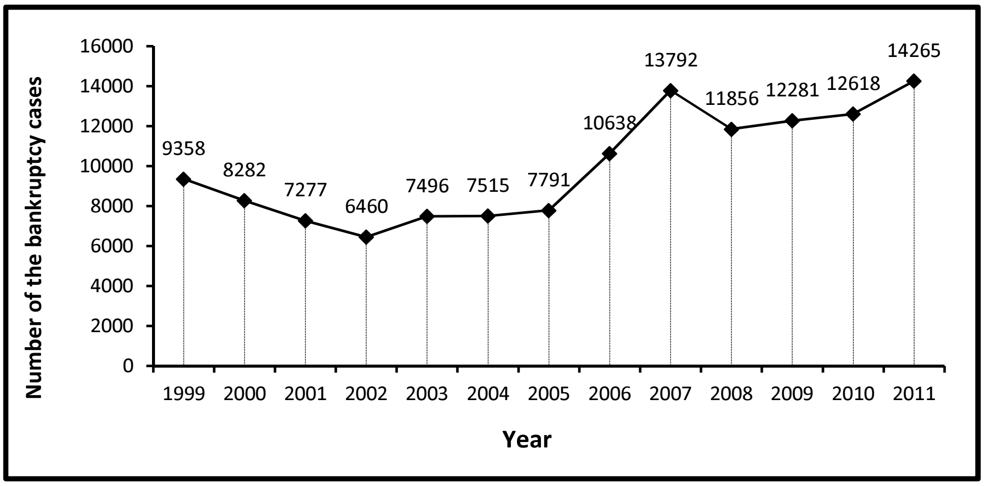
ESTIMATION OF ENTERPRISE'S FINANCIAL CRISIS THAT COULD LEAD TO ITS BANKRUPTCY AND MARKOV CHAINS

Statement of the problem

Negative trends in the global financial and economic recession led to the situation in which the problem of bankruptcy has become one of the most relevant in almost every country and Ukraine isn't an exception (indicators of bankruptcy cases are increasing every year. There were 6460 bankruptcy cases of enterprises of different ownership in 2002 whereas by 2011 this number was already 14265, figure 1) [4].



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The critical condition of the national economy as a whole and its individual sectors is the result of a number of internal and external factors. The decline in the production, lack of working capital, deterioration of the property potential cause a significant deterioration of the business.

The diagnostic of enterprise's crisis is regarded as one of the way of economic diagnostic and is closely related to the diagnostic of threat of bankruptcy. This forecast is based on the development of the multivariate regression models and takes into account the factors that has the most significant effect on the development and formation of the greatest threats of bankruptcy in the coming period.

The purpose

To identify the key factors in the integrated assessment of the bankruptcy, build a tree structure of the crisis factors of the Ukrainian enterprises based on the analysis of financial and economic indicators and highlight the possibility of using Markov chains to determine the average probability of bankruptcy.

The findings

Combined estimation of bankruptcy risk can be performed on the quantitative indicators (calculated with the help of Altman's method, Beaver's estimation, fuzzy neural nets, etc.). [2] and qualitative benefits of the scale and relat ionships between factors in the hierarchy of these factors. Determination of the bankruptcy indicator is based on the data from all levels of the hierarchy. The scheme of the process is described in the following mathematical model of bankruptcy risk (BRM-Bankruptcy Risk Model): [1]

$$BRB=\langle G, L, \Phi \rangle,$$

where

G - tree structure factors of bankruptcy;

L - the set of the qualitative assessments of levels of each factor;

Φ - system of the preference relation between factors within the same level of the hierarchy:

$$\Phi = \{ F_i(\varphi)F_j \mid \varphi \in (\succ, \approx) \},$$

where

$\succ$ - preference relation;

$\approx$ - indifference relation. [1]

It is necessary to select a set of indicators that would accurately assess the financial position of the company in terms of risk, profitability and financial stability.

For example, the depreciating of a fixed asset, the return on assets (ROA), market liquidity, accounts payable, etc are relevant indicators for the Ukrainian companies. [2] With all this in mind, we can construct an optimal tree hierarchy of the critical factors for national companies (Figure 2).

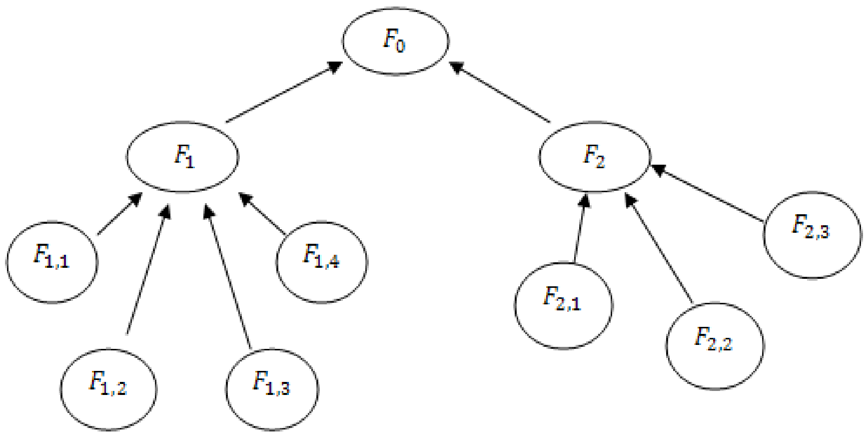

Figure 2. The tree hierarchy of estimation of the company's recession condition.

F_0 – the root node, corresponding to the risk factors of corporation on whole;
F_1 – indicator of the company's effectiveness;
F_2 – indicator of the financial stability;
$F_{1,1}$ – coefficient of the depreciation of fixed assets;
$F_{1,2}$ – fixed-asset turnover;
$F_{1,3}$ – the return on assets (ROA);
$F_{1,4}$ – coefficient of the assets' convertibility;
$F_{2,1}$ – liquidity ratio;
$F_{2,2}$ – correlation between borrowed funds and the company's equity;
$F_{2,3}$ – solvency ratio;

Each element in the hierarchy, as well as its every level, has a different degree of importance r_i, which is determined for each example.

Using the constructed hierarchy, we can count the operation for the assessment of the bankruptcy risk. Let event A be a decline in the company's effectiveness, event C - the reduction of the financial stability, then from the law of total probability we obtain:

$$P(A)=\sum_{j=1}^{4}P(F_{1,j})P(A\,|\,\mathrm{F}_{1,\mathrm{j}})$$

$$P(C)=\sum_{j=1}^{3}P(F_{2,j})P(C\mid F_{2,j})$$

where $P(A), P(B)$ – the probabilities of A and B,
$P(A|F_{1,j}), P(C|F_{2,j})$ – the conditional probabilities of A given $F_{1,j}$ and C given $F_{2,j}$.

Let B be an event that is the occurrence of bankruptcy of the company F_0, then we have:

$$P(B)=P(A)P(B\mid A)+P(C)P(B\mid C).$$

Next, we will show the possibility of using the stationary continuous-time Markov chains in predicting the possibility of bankruptcy. Let us recall some theoretical information.

Let our system S remain in the same state in every moment of time, forming a countable set $E=\{1, 2, \dots, n\}$. It is said that the system is described by Markov chains if the next state depends only on the current state and not on the sequence of events that preceded it.

Let $X(t)$ be the state of the system in the moment of the time t, then using the definition of discrete-time Markov chain we have:

$$P\{X(t)=j|\, X(1)=i_1,\dots,X(t-1)=i\}=$$
$$=P\{X(t)=j|\, X(t-1)=i\}=p_{ij}.$$

Continuous-time Markov chains differ from the discrete-time Markov chains becouse the transitions between states can occur at any moment of the time, and the next move is also a random variable.

Also, if the the probability of the transition is independent of t, then the chain is called time-homogeneous or stationary.

Let P be the transition matrix of Markov chain

$$P=\begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}$$

and after n steps we will get:

$$P^n=\begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix}^n$$

Suppose that we have the initial distribution of a Markov chain

$$p^{(0)}=\{p_i\,, i\in E\}, \qquad \sum_{i=1}^{n}p_i=1$$

then the distribution on the $(n+1)$ step will be determined by the following relation

$$p^{(n+1)}=p^{(0)}\,P^n.\ [3]$$

Therefore, the evolution of a Markov chain is fully completely described by the initial distribution and transition probability matrix.

Let us consider the limit of the probability of transition from state i to state j at time t, where $t \to \infty$. If with the increase in t, the influence of the state $X(t) = i$ becomes smaller

$$\lim_{t \to \infty} p_{ij}(t) = p_j.$$

then the chain is said to be ergodic.

Let a function $f(x(t))$ be the probability of bankruptcy that depends on the state in which the company is situated, which is defined on the set of states of a Markov chain that is highlighted below in a figure:

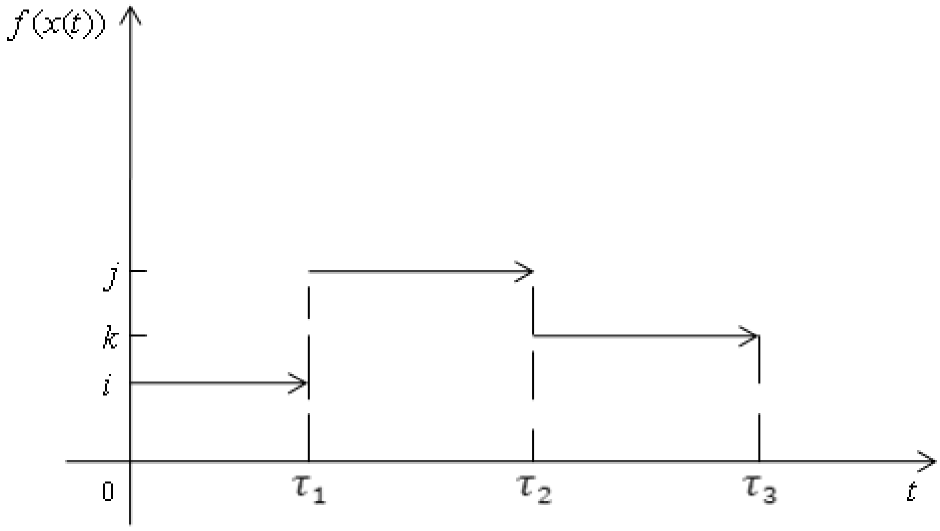

Figure 3. Example of behavior of the system S of Markov chain.

$\tau_1, \tau_2, \ldots, \tau_n -$ moments of going from one state to another; $\tau = \min_t \{x(t) \neq x(0)\} -$ moment of the transition from the initial state, $\tau_n = \min_{t > \tau_{n-1}} \{x(t) \neq x(\tau_{n-1})\}$. The distribution of random variable τ_i will be defined with such ratio:

$$P\{\tau_i \leq t\} = \lambda_i e^{-\lambda_i t}, \qquad t > 0$$

because, a Markov chain is the stochastic process that can be thought of as memoryless(Markov property), that's why the moments of state transition is the best described by the exponential distribution, because only this distribution has the simple property. Now we will demonstrate it.

Let the random variable ξ have exponential distribution with parameter ($\lambda > 0$). Then we obtain:

$$P\{\xi > n + m | \xi > n\} = P\{\xi > m\}, \qquad n, m > 0$$

Proof: $P\{\,\xi > n\,\} = \int_n^{\infty} \lambda e^{-\lambda x}\,dx = \,e^{-\lambda n}$

$$P\{\,\xi > n+m|\,\xi > n\,\} = \frac{P\{\xi > n+m\}}{P\{\xi > n\}} =$$

$$= \frac{e^{-\lambda(m+n)}}{e^{-\lambda n}} = \,e^{-\lambda m} = P\{\,\xi > m\}.$$

□

Thus, the system S is given by:

1. initial distribution $p^{(0)} = \{p_i\,, i\, \in E\}$;
2. vector of parameters of exponential distribution $\lambda = \{\lambda_i,\ i\, \in E\}$;
3. matrix $P = \{p_{ij}\,, i, j\, \in E\}$.[4]

We will also assume that our Markov chain is irreducible and the stationary distribution $\{p_1, p_2, \ldots, p_n\}$ exists, namely:

$$p_j \;=\; \sum_{s=1}^{n} p_s p_{sj}\,,$$

where $p_{sj} \;= P\{x(\tau) = j|x(0) = s\} -$ transition probabilities.

Now we will consider the average probability of bankruptcy $f(x(t))$ along the trajectory of the process, in other words the value:

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T f(x(t))dt.$$

According to the Markov ergodic theorem this variable exists and is defined by the following ratio:

$$\overline{f(x(t))} \;=\; \lim_{T\to\infty} \frac{1}{T}\int_0^T f\big(x(t)\big)dt \;=\; \frac{\sum p_i\, M_i(\int_0^{\tau} f(x(u))du)}{\sum p_i\, M_i\tau} =$$

$$= \frac{\sum p_i\, f(i) M_i\tau}{\sum p_i\, M_i\tau} \;=\; \frac{\sum p_i\, f(i)\lambda_i{}^{-1}}{\sum p_i\, \lambda_i{}^{-1}} \;=\; \frac{\sum p_i\, f_i\lambda_i{}^{-1}}{\sum p_i\, \lambda_i{}^{-1}}\,,$$

where $\tau -$ moment of the transition from the initial state (also has the exponential distribution).

Conclusions

The probability of bankruptcy is determined by many factors, such as the probability of the crisis factors and the level of dependence of the enterprise on these factors. Therefore it is necessary to develop a model in which the assessment of the financial conditions of Ukrainian enterprises would be based on national standards of accounting and reporting which would use statistical and information database of local businesses.

Since the position of the company at some moment of time is completely determined by the probabilistic characteristics of the next evolution of the system, we can assume that our system is described by a Markov chain. Using time-homogeneous Markov chains and Markov ergodic theorem, we can derive the average probability of bankruptcy along the the process.

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Abstract:

The probability of bankruptcy is determined by many factors, such as the probability of the crisis factors and the level of dependence of the enterprise on these factors. Therefore it is necessary to develop a model in which the assessment of the financial conditions of Ukrainian enterprises would be based on national standards of accounting and reporting which would use statistical and information database of local businesses.

Since the position of the company at some moment of time is completely determined by the probabilistic characteristics of the next evolution of the system, we can assume that our system is described by a Markov chain. Using time-homogeneous Markov chains and Markov ergodic theorem, we can derive the average probability of bankruptcy along the the process.

Estymacja wpływu kryzysu finansowego bankrutujących przdsiębiorstw w kontekście łańuchów Markowa

Problem bankructwa przedsiębiorstw jest jedną z ważniejszych kwestii we współczesnej gospodarce. Uwidacznia się on szczególnie w czasie kryzysów finansowych czy recesji. Również na Ukrainie problem ten występuje w coraz większej skali. W pracy podjęto próbę oszacowania przeciętnego prawdopodobieństwa bankructwa w gospodarce ukraińskiej w zależności od oddziałujących czynników za pomocą modelu opartego na łańcuchach Markowa, które stają się coraz częstszym narzędziem przy badaniu zjawisk i procesów ekonomicznych i finansowych.

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